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# Stochastic loss reserving for General Insurance: a Gaussian approach



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Introduction	Multivariate normal distribution	Setting the parameters	Paid - incurred model	Portfolio model	Conclusion
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## Stochastic loss reserving for General Insurance: a Gaussian approach

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Pp

## Basic goal of loss reserving

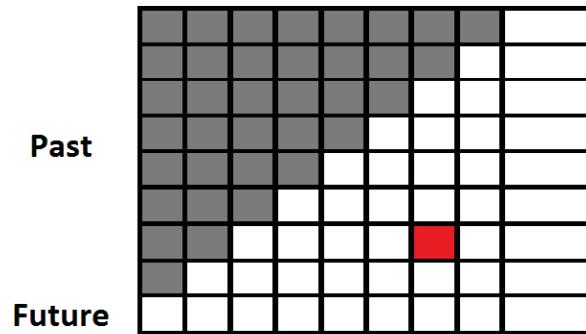


Figure : What will the amount (loss, incurred) be in the red cell?

Pp

## Motivation for stochastic loss reserving

- ▶ Capital optimization (portfolio, reinsurance, and pricing)
- ▶ IFRS/IAS valuation
- ▶ Solvency requirements

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## Limitations of chain ladder

Chain ladder ...

- ▶ Performs poorly on longer tailed LOBs  
Additional assumptions needed for development factors of later development periods
- ▶ Is not able to model trends in any direction  
No application of actuarial knowledge possible
- ▶ Is not consistent in its predictions  
Calculate the loss reserve for a loss period at once or as the sum of individual cells?
- ▶ Is a deterministic method  
Bootstrap allows us to generate desired percentiles, but does not improve our model!

Pp

# Multivariate normal distribution

Pp

## Basic model assumptions

Let  $Y$  denote all (known and unknown) cells of a runoff table.  
Each LOB can be modeled by:

$$Y \sim \mathcal{N}(\mu, \Sigma)$$

The term *runoff table* can mean:

- ▶ incremental paid
- ▶ incremental incurred
- ▶ ...

Pp

## Properties

If  $Y \sim \mathcal{N}(\mu, \Sigma)$  then ...

### 1. Closed under linear transformations

$$SY \sim \mathcal{N}(S\mu, S\Sigma S^T) \text{ for a matrix } S$$

In practice ...

- ▶ aggregation of data
- ▶ aggregation of predictions
- ▶ discounting future payments

Pp

## Properties

### 2. Closed under conditioning

$$S_2 Y | \{S_1 Y = s_1 y\} \sim \mathcal{N}(, ) \text{ for matrices } S_1 \text{ and } S_2$$

In practice ...

- ▶ prediction
- ▶ to add information (to be discussed in paid-incurred model)

### 3. Able to handle negative incremental losses

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## Aggregation of data

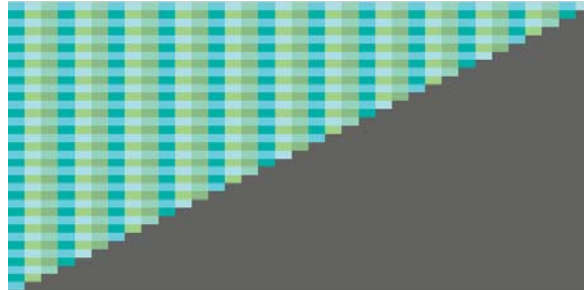


Figure : Suppose you have quarterly data

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## Aggregation of data

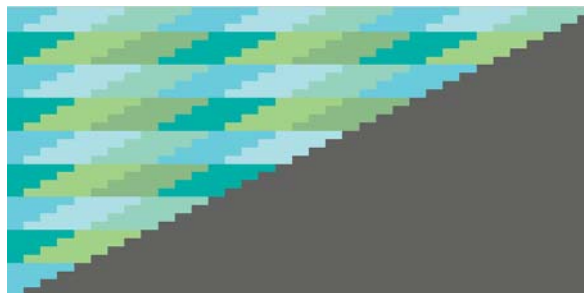


Figure : Aggregate into yearly observations

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## Aggregation of data



Figure : Aggregate the older development periods

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## Aggregation of predictions

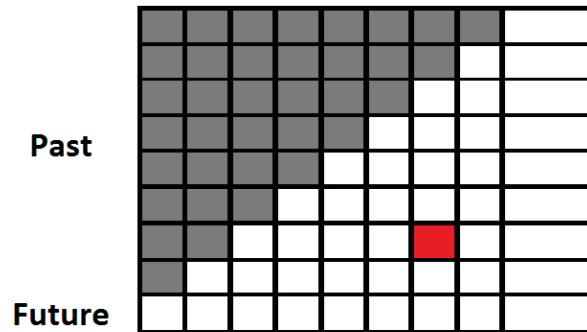


Figure : We can predict one future payment

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## Aggregation of predictions

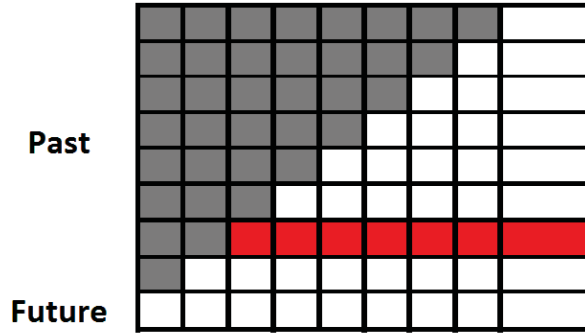


Figure : Or the sum of future payments for a loss period

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## Aggregation of predictions

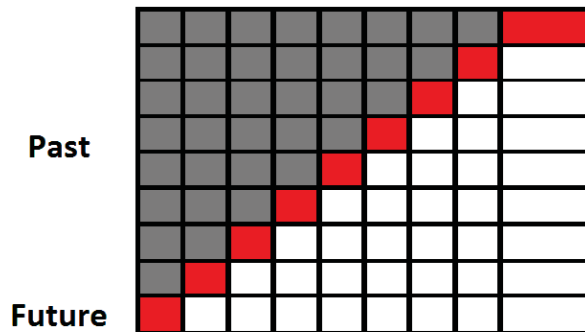


Figure : Or the sum of payments for a future booking or accounting period

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# Mean and covariance matrix

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## Main ingredients for mean and covariance matrix 1

We need to find expressions for the elements in  $\mu$  and  $\Sigma$ , taking into account:

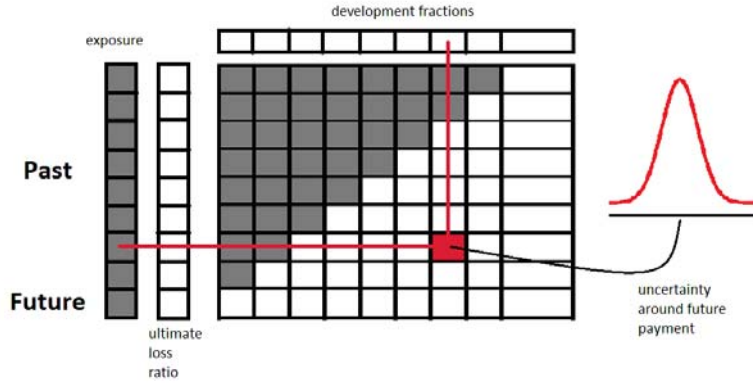
1. exposure per loss period
2. ultimate loss ratio per loss period
3. development fraction per development period

Instead of asking . . . what amount do we expect to pay over a given loss period in some development period?

We ask . . . what **fraction** of the **ultimate loss** of a loss period do we expect to pay in a development period?

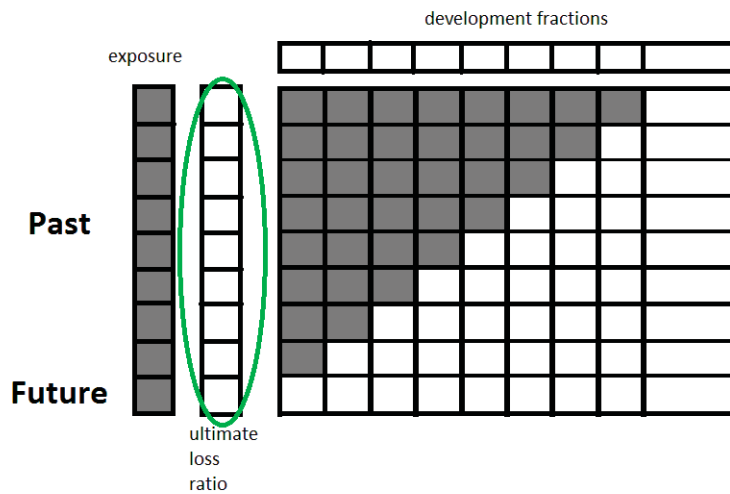
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## Main ingredients for mean and covariance matrix 2



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## Ultimate loss ratios 1



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## Ultimate loss ratios 2

We choose a trend of the ultimate loss ratios over the loss periods . . .

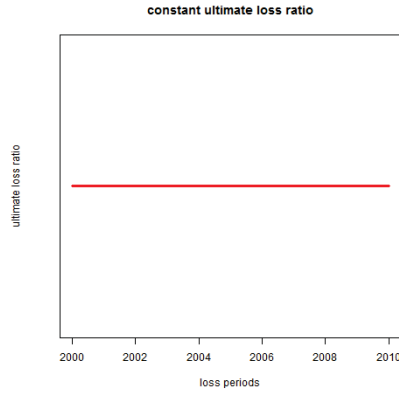


Figure : The ultimate loss ratios are constant over the loss periods

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## Ultimate loss ratios 2

We choose a trend of the ultimate loss ratios over the loss periods . . .

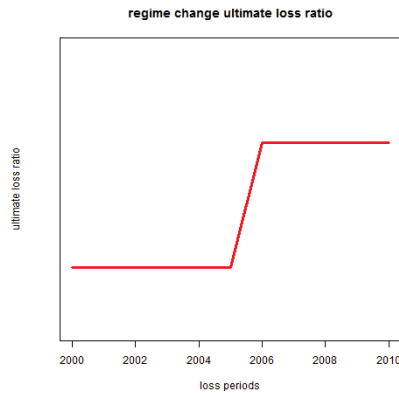


Figure : There is a jump in the ultimate loss ratios at a certain loss period

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## Ultimate loss ratios 2

We choose a trend of the ultimate loss ratios over the loss periods . . .

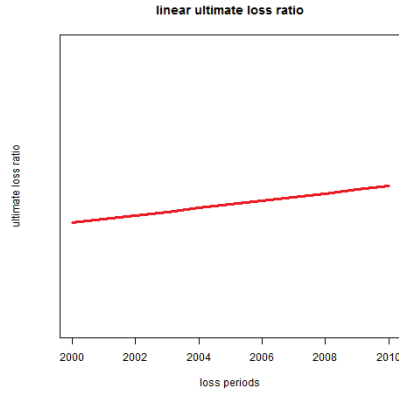


Figure : The ultimate loss ratios are increasing at a constant rate

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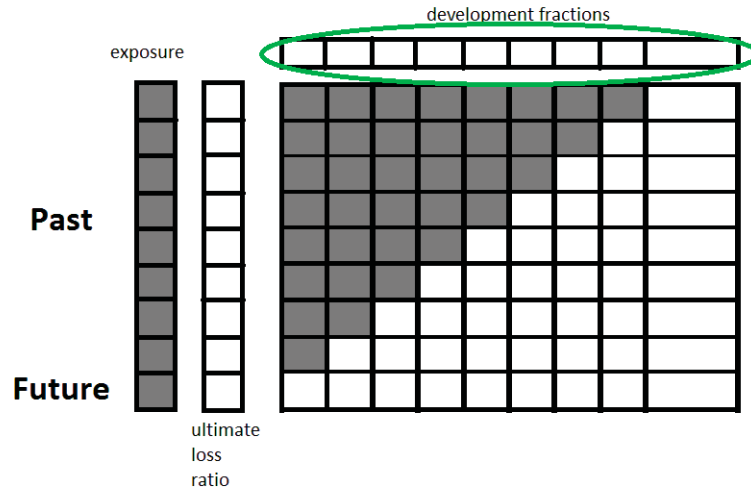
## Ultimate loss ratios 3

We choose a trend of the ultimate loss ratios over the loss periods . . .

1. Together with exposure, we are able to model future loss periods
2. Reduce the number of parameters to estimate
3. Application of actuarial judgment

Pp

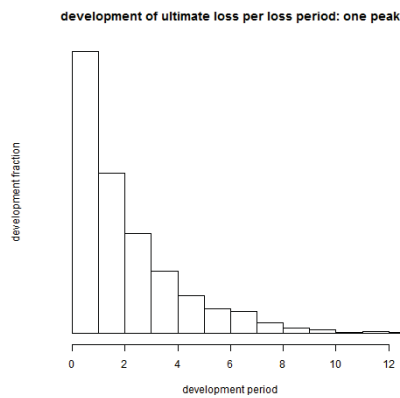
## Development fractions 1



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## Development fractions 2

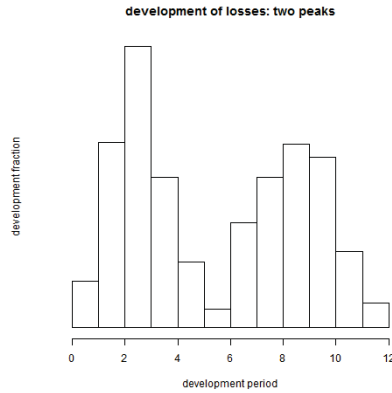
In each development period we have to pay a fraction of the ultimate loss of the corresponding loss period.



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## Development fractions 2

In each development period we have to pay a fraction of the ultimate loss of the corresponding loss period.



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## Development fractions 3

Instead of choosing a fraction for each development period, we choose a curve for the development of the losses

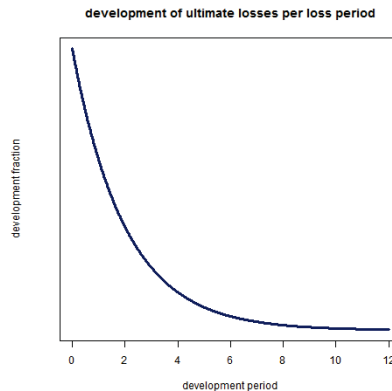


Figure : Losses settle quickly

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## Development fractions 3

Instead of choosing a fraction for each development period, we choose a curve for the development of the losses

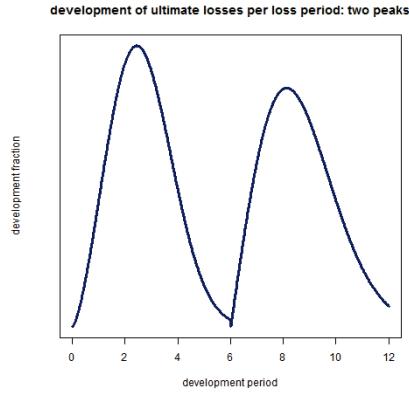


Figure : Two peaks in the development

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## Development fractions 4

We choose a curve for the development of the losses . . .

1. Long-tailed businesses
2. Reduce the number of parameters to estimate
3. Application of actuarial judgment

Pp

The elements in  $\mu$  and  $\Sigma$  (loss period  $l$ , development period  $k$ ) are functions of:

1. exposure per loss period:  $w_l$
2. ultimate loss ratio per loss period:  $(X\beta)_l$ , where  $X$  is the chosen trend of the loss ratios  $\beta$  are the parameters to be estimated
3. development fraction per development period:  $\pi_{l,k}$   

$$\pi_{l,k} = \int_0^1 \int_{\max(t,k-1)}^k \frac{1}{\alpha_l} b\left(\frac{s-t}{\alpha_l}; \theta\right) ds dt$$
 $b(\cdot; \theta)$  is the chosen curve  
 $\alpha_l$  to model development regimes

Pp

## Paid - incurred model

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## Motivation

Suppose we have ...

- ▶ An incremental paid run-off table  $Y^{(p)}$ , and
- ▶ An incremental incurred (incremental paid + changes in case reserves) run-off table  $Y^{(i)}$
- ▶ No idea whether to use the paid or the incurred triangle to set our reserves

We want to ...

- ▶ Use the knowledge of the claims handling department
- ▶ Use both triangles in setting our reserves
- ▶ Create a joint model for  $Y = (Y^{(p)}, Y^{(i)})$

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## Auxiliary variables

We start with auxiliary independent Gaussian random variables:  $Z_{lk}^{(i)}$  and  $Z_{lk}^{(p)}$

$$R = \left\{ \sum_k Z_{lk}^{(i)} = \sum_k Z_{lk}^{(p)} \quad (\forall l) \right\}$$

This says that for each loss period, the total amount incurred equals the total amount paid.

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## Final step

Finally we define the incremental losses by

$$Y^{(i)} \sim Z^{(i)} | R$$

$$Y^{(p)} \sim Z^{(p)} | R$$

This means that  $(Y^{(1)}, Y^{(2)})$  is normally distributed, and that the row sums of the two tables are always equal.

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## Example

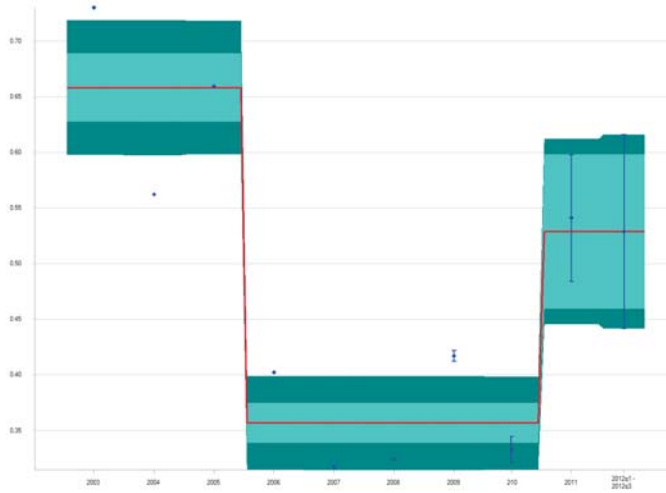
We consider three lines of business

- ▶ Fire Real Estate  
Loss periods: 2003q1 - 2011q3  
Future loss periods: 2011q4 - 2012q3
- ▶ Third Party Liability Motor  
Loss periods: 1998q1 - 2011q3  
Future loss periods: 2011q4 - 2012q3
- ▶ Liability  
Loss periods: 2004q1 - 2011q3  
Future loss periods: 2011q4 - 2012q3

Exposure for future loss year: \$ 10,000

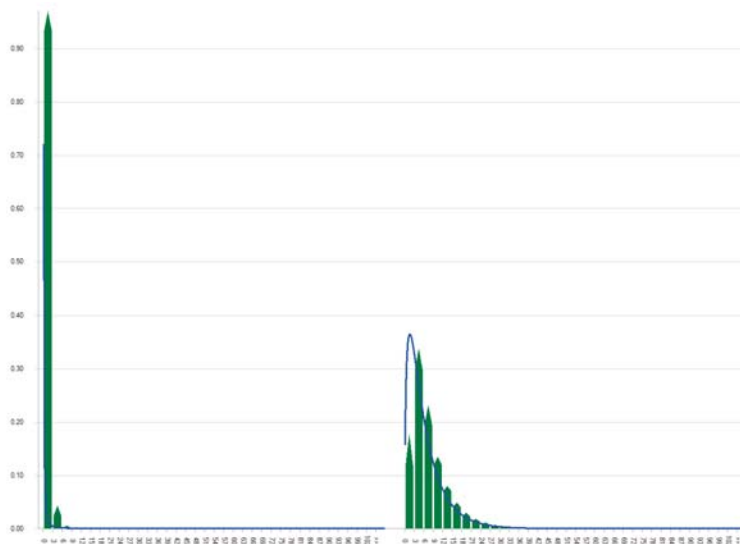
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## Fire Real Estate



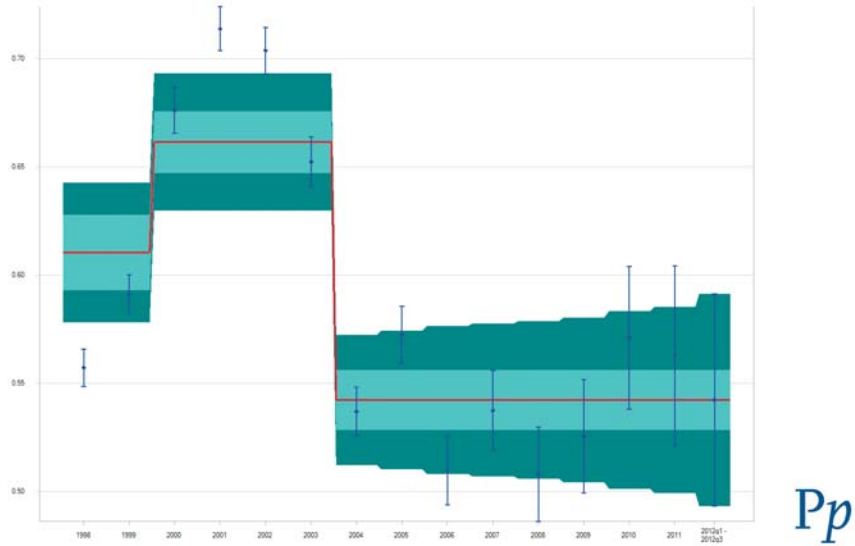
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## Fire Real Estate



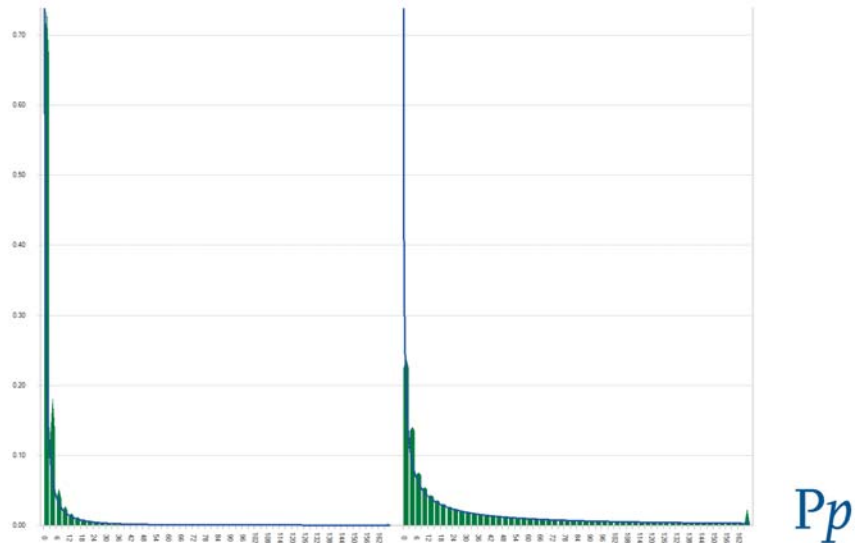
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## TPL Motor



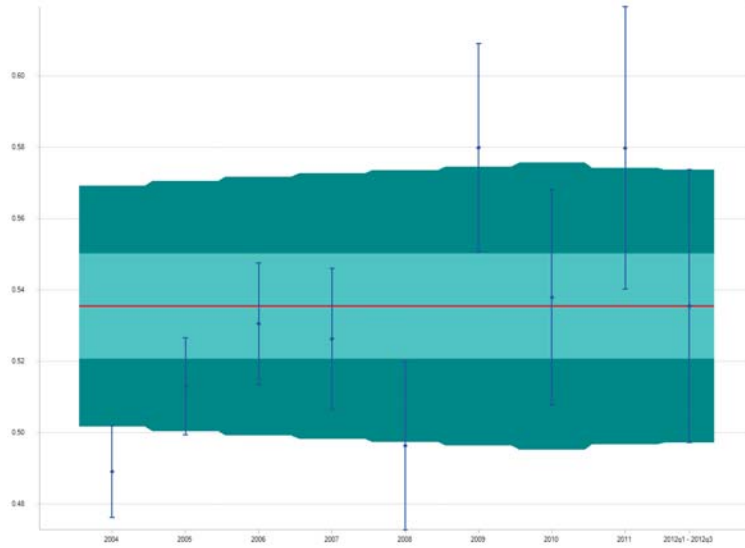
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## TPL Motor



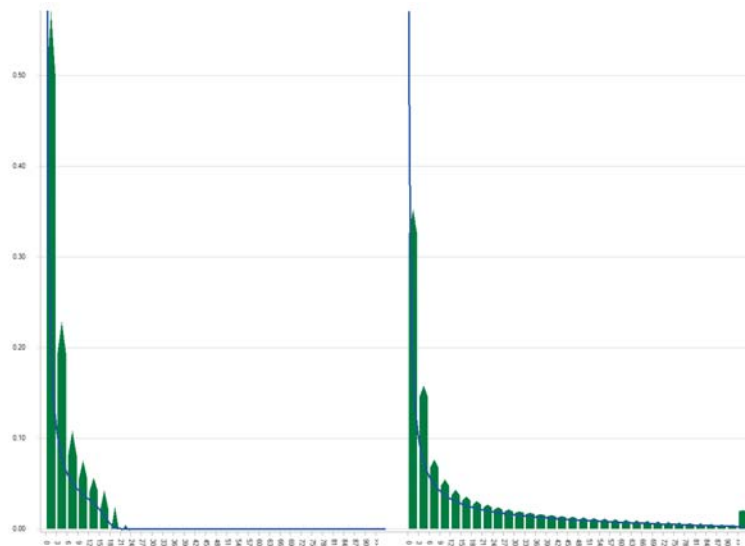
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## Liability



$P_p$

## Liability



$P_p$

## Loss reserves and IBNR

	80th quantile	IBNR
Total (sum)	49,724	20,316
Fire Real Estate	8,959	5,871
TPL Motor	28,945	8,302
Liability	11,821	6,143

Results include future loss periods

Pp

## Portfolio model

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## Motivation

Suppose we have . . .

- ▶ multiple lines of business in our portfolio
- ▶ a model for each LOB (possibly paid-incurred models)
- ▶ perhaps an ad-hoc approach for diversification

We want to . . .

- ▶ model the dependencies between these LOBs, for . . .
- ▶ better predictions
- ▶ more accurate assessment of capital requirements

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Let  $Y_r$  ( $r = 1, \dots, R$ ) denote all (known and unknown) cells of a runoff table of LOB  $r$  in a portfolio. Each LOB can be modeled by:

$$Y_r \sim \mathcal{N}(\mu_r, \Sigma_r)$$

The term *runoff table* can mean:

- ▶ incremental paid
- ▶ incremental incurred
- ▶ incremental paid and incurred

Pp

## Problem

The goal is to find mean  $\mu$  and covariance  $\Sigma$  of:

$$Y = (Y_1^\top, \dots, Y_R^\top)^\top \sim \mathcal{N}(\mu, \Sigma)$$

$$\mu = (\mu_1^\top, \dots, \mu_R^\top)^\top$$

$$\Sigma = \begin{bmatrix} \Sigma_1 & \dots & \Sigma_{1R} \\ \vdots & \ddots & \vdots \\ \Sigma_{R1} & \dots & \Sigma_R \end{bmatrix}$$

Pp

## Solution

- ▶ For each  $\Sigma_r$  there exists an unique symmetric matrix  $B_r$  such that

$$B_r B_r = \Sigma_r$$

$$B_r = J D_r^{\frac{1}{2}} J^\top$$

eigendecomposition

- ▶ Cross covariance matrix

$\Sigma_{rr^*} = \rho_{rr^*} B_r B_{r^*}$
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- ▶  $\rho_{rr^*}$  is an element of the  $R \times R$  correlation matrix (symmetric, positive definite).

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## Solution

$$\Sigma = \begin{bmatrix} \Sigma_1 & \dots & \rho_{1R} B_1 B_R \\ \vdots & \ddots & \vdots \\ \rho_{1R} B_R B_1 & \dots & \Sigma_R \end{bmatrix}$$

Pp

## Example

The portfolio consists of:

- ▶ Fire Real Estate
- ▶ Third Party Liability Motor
- ▶ Liability

### Correlation matrix

	Fire Real Estate	TPL Motor	Liability
Fire Real Estate	1	-.17	.53
TPL Motor		1	.42
Liability			1

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## Loss reserves and IBNR

	80th quantile	IBNR	Diversification gain
Total	48,355	18,946	1,370
Fire Real Estate	8,329	5,242	630
TPL Motor	28,579	7,935	366
Liability	11,447	5,769	374

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## Conclusions and take-home messages

1. Both Solvency II and IFRS/IAS ask for a full stochastic model of future payments
2. IFM is such a model for Loss Reserving. As software it is compatible with all standard ICT environments. It efficiently produces all the necessary market-value based outcomes, which are fully in compliance with accounting standards and regulation requirements.
3. IFM produces the necessary information for managing profitability bottom up, thanks to the future risk approach. Portfolio optimization therefore becomes feasible.
4. Last but not least: with the use of IFM a maximum of 2 professionals is needed for the relevant actuarial Solvency II, IFRS/IAS functions.

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## Answers to questions like

- ▶ What are the right premiums for my portfolio given claims expectations? Also for (sub)branches, market segments, and homogeneous risk groups.
- ▶ How to improve my portfolio profitability?
- ▶ What will be the claims levels - both in terms of cash-out and reserving - for the next few years under present company policy? How can I manage these better?
- ▶ What are my insurance risks and what can I do about them?

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## ...and

- ▶ How do I get insight in claims reserves? How can I be sure that I have met all requirements?
- ▶ How can I better test and determine my reinsurance requirements?
- ▶ And last but not least: what is the minimal (internal) professional staffing and expertise required to address all the above questions? And to what extent can intelligent software reduce these?

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## The software and the people of Posthuma Partners offer a practical solution to all these questions

- ▶ Structural and permanent insight in the portfolio with regard to risk profile, claims and loss control (cash-out and reserve) and required premium setting - available in a modular way for various levels
- ▶ High quality projections of claims development for the coming years for optimal working capital management and external reporting
- ▶ Active control of insurance risks at all levels



## ... and perhaps even more importantly:

- ▶ Management information, dashboard with control buttons and scenario calculations for adjustments
- ▶ With the use of IFM a maximum of 2 professionals is required for the relevant actuarial reporting functions



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